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 $V_{ud}$  from Nuclear DecaysI.S. TOWNER AND J.C. HARDY<sup>1</sup>*Cyclotron Institute, Texas A & M University,  
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The current value of  $V_{ud}$  is determined from superallowed beta-decay experiments. Other methods, briefly summarised here, have to overcome specific experimental hurdles before they are competitive. However, the nuclear results do depend on a nuclear-structure calculation of isospin-symmetry breaking, which is often a cause of some concern. We show here, by adopting the Conserved Vector Current (CVC) hypothesis, that these theoretical corrections can be tested for consistency. In this test, calculations based on the shell model with Saxon-Woods radial functions perform the best.

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# 1 Current value for $V_{ud}$

Superaligned  $0^+ \rightarrow 0^+$  beta decay between isospin  $T = 1$  nuclear analog states currently provides the most accurate determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element,  $V_{ud}$ . There are two reasons for this: First, there are many nuclear beta decays that could be chosen for study. By limiting the study to just those decays between  $0^+$  analog states only the vector component of the weak interaction is operative. Second, by limiting the study in this way, the Conserved Vector Current (CVC) hypothesis becomes useful. This hypothesis states that the strength of the vector component of the weak interaction,  $G_V$ , is a ‘true’ constant and independent of the nucleus under study. This result provides a consistency check among the different nuclear decays studied. The hypothesis, however, is only operative in the isospin-symmetry limit. So one disadvantage is that a nuclear-structure dependent calculation of isospin-symmetry breaking is required and the uncertainty associated with this is the subject of the second half of this report. For the moment, we note that this correction is small and is testable via the CVC hypothesis.

The analysis proceeds as follows. The experimental measured quantities of  $Q$ -value, lifetime and branching ratio are combined into an  $ft$  value. To this, radiative and isospin-symmetry breaking corrections are added in defining a ‘corrected’  $\mathcal{F}t$  value

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}. \quad (1)$$

Here the radiative correction is separated into three terms,  $\delta'_R$ ,  $\delta_{NS}$  and  $\Delta_R^V$ , where  $\delta'_R$  and  $\delta_{NS}$  depend on the nucleus under study, while  $\Delta_R^V$  does not. Further,  $\delta'_R$  depends trivially on the nucleus, depending on the total charge of the nucleus,  $Z$ , and on the emitted electron’s energy. But  $\delta_{NS}$ , like the isospin-symmetry breaking correction  $\delta_C$  depends in its evaluation on the details of nuclear structure. Lastly,  $K$  is a constant,  $K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2 / (m_e c^2)^5$ , with  $m_e$  the electron mass. Immediately one sees from the CVC hypothesis that a nucleus-independent  $G_V$  value leads to the requirement that the  $\mathcal{F}t$  value be nucleus-independent as well. This provides a demanding consistency check on the experimental measurements and the theoretical corrections. If the  $\mathcal{F}t$  values are found to be statistically consistent with each other, then one is justified in taking an average value,  $\overline{\mathcal{F}t}$ , from which  $G_V$  can be determined. In addition, via the relationship  $V_{ud} = G_V/G_F$ , where  $G_F$  is the well-known weak-interaction strength constant for a purely leptonic decay, a value for  $V_{ud}$  is obtained as well.

From the 2009 survey of experimental data, Hardy and Towner [1] determine the value of  $\overline{\mathcal{F}t}$  to be

$$\mathcal{F}t = 3071.81 \pm 0.83 \text{ s} \quad (2)$$

leading to

$$|V_{ud}| = 0.97425 \pm 0.00022. \quad [0 \rightarrow 0] \quad (3)$$

Other methods of obtaining  $V_{ud}$  – see survey in [2] – are currently less accurate. They are:

- *neutron decay*, for which

$$|V_{ud}| = 0.9743 \pm 0.0015. \quad [\text{neutron}] \quad (4)$$

This value was presented at the CKM2010 Workshop by Märkisch [3]. It is based on the 2010 Particle Data Group’s analysis [4], but updated for new lifetime measurements from Serebrov *et al.* [5] and Pichlmaier *et al.* [6] and for preliminary beta-asymmetry measurements from PERKEO II [7] and UCNA [8].

- $T = 1/2$  *mirror transitions*, for which

$$|V_{ud}| = 0.9719 \pm 0.0017 \quad [\text{mirror transitions}] \quad (5)$$

from Naviliat-Cuncic and Severijns [9].

- *pion beta decay*, for which

$$|V_{ud}| = 0.9742 \pm 0.0026 \quad [\text{pion}] \quad (6)$$

using the branching ratio measured by the PIBETA group [10].

The CKM matrix is posited to be unitary. To date, the most demanding test of this comes from the sum of squares of the top-row elements,  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ , which should sum to one. Taking  $|V_{us}|$  from the recent FlaviaNet report [11],  $|V_{us}| = 0.2253(9)$ , and  $|V_{ub}|$  from the Particle Data Group [4],  $|V_{ub}| = 3.39(44) \times 10^{-3}$ , the unitarity sum becomes

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99990 \pm 0.00060. \quad (7)$$

This result shows unitarity to be fully satisfied to a precision of 0.06%. Only  $V_{us}$  and  $V_{ud}$  contribute perceptibly to the uncertainty and their contributions to the error budget are almost equal to one another.

## 2 Test of isospin-symmetry breaking correction

Let’s return to the isospin-symmetry breaking correction,  $\delta_C$ . Its evaluation requires a nuclear-structure calculation. Although the role played by nuclear structure is relatively small, the precision currently reached by experiment is such that the theoretical uncertainties introduced with  $\delta_C$  now dominate over the experimental uncertainties. Consequently, this correction has attracted a lot of attention recently. We

offer a recommended set of values for this correction [12, 1], but there are a growing number of alternative choices [13, 14, 15, 16, 17]. There has also been a claim, albeit unsupported by any detailed computations, that our calculations neglect a radial excitation term, which is purported to be important [18]. To counterbalance that, however, there are two recent papers that confirm our result: one [19] does so based on a semi-empirical analysis of the data, while the other [20] quotes results from a Skyrme-density-functional-theory calculation in which simultaneous isospin and angular-momentum projection has been incorporated.

Clearly it would be valuable if the various sets of  $\delta_C$  corrections could be tested against the experimental data. Towner and Hardy [21] have suggested such a test, which is based on the acceptance of the CVC hypothesis. We start by rearranging Eq. (1) to read

$$\delta_C = 1 + \delta_{NS} - \frac{\overline{\mathcal{F}t}}{ft(1 + \delta'_R)} \quad (8)$$

where  $\mathcal{F}t$  has been replaced by its average value. For any set of  $\delta_C$  values to be acceptable, this equation must be satisfied. For a series of  $n$  superallowed transitions, one treats  $\overline{\mathcal{F}t}$  as a single adjustable parameter and use it to bring the  $n$  results from the right-hand side of Eq. (8), which is based predominantly on the experimental  $ft$  values ( $\delta_{NS}$  is small, and  $\delta'_R$  unambiguous), into the best possible agreement with the corresponding  $n$  calculated values for  $\delta_C$ . The normalized  $\chi^2$ , minimized by this process then provides a figure of merit for that set of calculations.

The recent  $\delta_C$  calculations are described in [21] and are:

- Shell model with Saxon-Woods radial functions, SM-SW [12].
- Shell model with Hartree-Fock radial functions, SM-HF [1].
- Relativistic Hartree-Fock with the random phase approximation (RPA) and an effective interaction labelled PKO1, RHF-RPA [16].
- Relativistic Hartree with RPA and a density-dependent effective interaction, labelled DD-ME2, RH-RPA [16].
- Isovector monopole resonance model, IVMR [17].

We have applied the test to these five sets of model calculations. The resulting normalized  $\chi^2$  for each least-squares fit – expressed as  $\chi^2/n_d$ , where  $n_d$  is the number of degrees of freedom – is given in Table 1.

We give three sets of normalized  $\chi^2$ ; they differ one from another on how the uncertainties are handled. Strictly speaking, the  $\chi^2$  test only has an unambiguous interpretation if the errors considered are solely statistical. Thus in the first row in Table 1, we keep only statistical errors on the experimental  $ft$  values and assign no

	SM-SW	SM-HF	RHF-RPA	RH-RPA	IVMR
$\chi^2/n_d$ (Row 1 – see text)	1.2	8.3	7.2	6.0	48.0
Confidence Level (%)	26	0	0	0	0
$\chi^2/n_d$ (Row 3 – see text)	0.4	2.2	2.7	2.1	11.0
$\chi^2/n_d$ (Row 4 – see text)	0.3	1.1	1.6	1.3	4.5

Table 1: Normalized  $\chi^2/n_d$  obtained in the test described in the text for five sets of model calculations of  $\delta_C$ . From Ref. [21].

errors to the theoretical quantities  $\delta'_R$ ,  $\delta_{NS}$  and  $\delta_C$ . For this case, we can define a confidence level as [4]

$$CL = \int_{\chi_0^2}^{\infty} P_{n_d}(\chi^2) d\chi^2 \quad (9)$$

where  $P_{n_d}(\chi^2)$  is the  $\chi^2$  probability distribution function for  $n_d$  degrees of freedom, and  $\chi_0^2$  is the minimum value of  $\chi^2$  obtained in the fit for the particular model set of values for  $\delta_C$ . Loosely speaking, the larger the value of  $CL$  the more acceptable are the values of  $\delta_C$  in satisfying the CVC hypothesis. The  $CL$  values are given in the second row of the Table. In the third row, we have added non-statistical errors to the radiative correction, while in the fourth row non-statistical errors are included for both the radiative and isospin-symmetry breaking corrections. The inclusion of non-statistical errors generally reduces the normalized  $\chi^2$  of the fit, but the ranking of the models remains unaltered.

The most obvious outcome of these analyses is that only one model, SM-SW, produces satisfactory agreement with CVC. All the others have confidence levels below 0.5%. It is somewhat surprising that SM-HF with Hartree-Fock radial functions does not do as well as Saxon-Woods radial functions. The problem is that these SM-HF calculations fail to give large enough  $\delta_C$  values for high- $Z$  cases of  $^{62}\text{Ga}$  and  $^{74}\text{Rb}$ . This has been noted before by Ormand and Brown [14]. We have tried varying the Skyrme interaction used in the Hartree-Fock calculation – to date we have sampled 12 interactions – but they all fail in the high- $Z$  cases. The discrepancy appears inherent in the SM-HF model.

## References

- [1] J. C. Hardy and I. S. Towner, Phys. Rev. C **79**, 055502 (2009).
- [2] I. S. Towner and J. C. Hardy, Rep. Prog. Phys. **73**, 046301 (2010).

- [3] B. Märkisch, invited talk at CKM2010: 6<sup>th</sup> Int. Workshop on the CKM Unitarity Triangle (Warwick) (2010).
- [4] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
- [5] A. Serebrov *et al.*, Phys. Lett. B **605**, 72 (2005).
- [6] A. Pichlmaier, V. Varlamov, K. Schreckenbach and P. Geltenbort, Phys. Lett. B **693**, 221 (2010).
- [7] H. Abele, Prog. Part. Nucl. Phys. **60**, 1 (2008).
- [8] J. Liu *et al.* for the UCNA Collaboration, arXiv:1007.3790 (2010).
- [9] O. Naviliat-Cuncic and N. Severijns, Phys. Rev. Lett. **102**, 142302 (2009).
- [10] D. Počanić *et al.*, Phys. Rev. Lett. **93**, 181803 (2004).
- [11] M. Antonelli *et al.* for the FlaviaNet Kaon Working Group, Eur. Phys. J. C **69**, 399 (2010).
- [12] I. S. Towner and J. C. Hardy, Phys. Rev. C **77**, 025501 (2008).
- [13] W. E. Ormand and B. A. Brown, Nucl. Phys. **A440**, 274 (1985); W. E. Ormand and B. A. Brown, Phys. Rev. Lett. **62**, 866 (1989).
- [14] W. E. Ormand and B. A. Brown, Phys. Rev. C **52**, 2455 (1995).
- [15] H. Sagawa, N. Van Giai and T. Suzuki, Phys. Rev. C **53**, 2163 (1996).
- [16] H. Liang, N. Van Giai and J. Meng, Phys. Rev. C **79**, 064316 (2009).
- [17] N. Auerbach, Phys. Rev. C **79**, 035502 (2009).
- [18] G. A. Miller and A. Schwenk, Phys. Rev. C **78**, 035501 (2008); G. A. Miller and A. Schwenk, Phys. Rev. C **80**, 064319 (2009).
- [19] G. F. Grinyer, C. E. Svensson and B. A. Brown, Nucl. Instrum. Methods A **622**, 236 (2010).
- [20] W. Satula, J. Dobaczewski, W. Nazarewicz and M. Rafalski, arXiv:1010.3099 (2010); W. Satula, J. Dobaczewski, W. Nazarewicz, M. Borucki and M. Rafalski, arXiv:1010.5053 (2010).
- [21] I. S. Towner and J. C. Hardy, Phys. Rev. C **82**, 065501 (2010).